

## Algebra II

## Notes 5.1 and 5.2: Rational exponents

Obj: Relate Roots and Rational exponents and use them to simplify expressions and solve equations

Quick review: Properties of exponents:

	Property	Example
Product:	$a^n \cdot a^m = a^{n+m}$	$x^2 \cdot x^3 = x^5$
Quotient:	$\frac{a^n}{a^m} = a^{n-m}$	$\frac{x^4}{x^3} = x^1 = x$
Power to power:	$(a^n)^m = a^{nm}$	$(x^2)^3 = x^6$
Power of Product /quotient:	$(a^n b^m)^p = a^{np} b^{mp}$	$(x^2 y^3)^4 = x^8 y^{12}$
Negative:	$a^{-n} = \frac{1}{a^n}$ $\left(\frac{a^n}{b^m}\right)^p = \frac{a^{np}}{b^{mp}}$	$x^{-3} = \frac{1}{x^3}$ $4^{-2} = \frac{1}{4^2}$

Another way to write a radical expression is with a rational (fraction) exponent.

$$25^{\frac{1}{2}} = \sqrt{25}$$

$$27^{\frac{1}{3}} = \sqrt[3]{27}$$

$$16^{\frac{3}{4}} = 4\sqrt[4]{16^3} \text{ or } \left(\sqrt[4]{16}\right)^3$$

In general:  $x^{\frac{1}{n}} = \sqrt[n]{x}$

denominator  
is root

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$$

or  $\sqrt[n]{x^m}$

numerator is power of the answer

**Example 1.** Rational exponents. Write the following as radicals.

$$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$$

$$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$$

$$32^{\frac{2}{5}} = (\sqrt[5]{32})^2 = 2^2 = 4$$

**Example 2.** Evaluate the expressions:

A.)  $32^{\frac{3}{5}} = (\sqrt[5]{32})^3 = 2^3 = 8$

$$27^{-\frac{2}{3}} = (\sqrt[3]{27})^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$50^{\frac{3}{4}} = (\sqrt[4]{50})^3 \rightarrow \text{show on calculator} = 18.803$$

You try.  $-(16^{\frac{3}{4}}) = -(\sqrt[4]{16})^3 = -(2)^3 = -8$

**Example 3.** Try these. Use your properties of exponents. Watch your bases. Simplify each expression in radical form.

a.  $81^{\frac{5}{6}} \cdot 81^{-\frac{1}{3}}$  *add*

$$\frac{5}{6} + \frac{-1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$81^{\frac{1}{2}} = 9$$

b.  $\left(\frac{xy^3}{x^{\frac{1}{2}}}\right)^{\frac{2}{3}}$  *mult.*

$$\frac{x^{\frac{2}{3}} y^2}{x^{\frac{1}{3}}} = x^{\frac{1}{3}} y^2$$

You try: a.  $\left(\frac{3}{32^{1/5}}\right)^{1/2}$

$$\frac{3^{1/2}}{32^{1/5}} = \frac{3^{1/2}}{2}$$

$${}^5\sqrt{32} = 2$$

b.  $2a^{1/3}(ab^{1/2})^{2/3}$

$$2a^{1/3}(a^{2/3}b^{1/3}) = 2ab^{1/3}$$

Extra practice: Using properties of exponents to simplify first.

$$x^2 \cdot x^3$$

$$\frac{1}{2} + \frac{2}{3}$$

$$\frac{x^2}{x^5}$$

$$2^{-3/5}$$

$$(16y^{-8})^{3/4}$$

$$(\sqrt[4]{16})^{-3}$$

$$x^{7/6}$$

$$x^{7/5}$$

$$16^{-3/4} y^6$$

$$2^{-3} y^6$$

$$\frac{y^6}{8}$$

or

$$x \cdot x^{1/6}$$

$$(8x^{15})^{-1/3}$$

$$\frac{x^2}{x^3}$$

$$\frac{4^2 - 1/3}{3/6 - 2/6}$$

$$x^{4/5} \cdot x^{5/2}$$

$$\frac{4}{5} + \frac{5}{2}$$

$$8^{-1/3} x^{-5}$$

$$(\sqrt[3]{8})^{-1}$$

$$x^{1/6}$$

$$x^{33/10}$$

$$\frac{8+25}{10}$$

$$\frac{33}{10}$$

$$\frac{2^{-1}}{x^5}$$

or

$$x^3 \cdot x^{3/10}$$

$$3^{3/10}$$

$$\frac{1}{2x^5}$$

